Figure 1: 1 2

1 Mathematics 433 Fall 2000

1.1 Final Exam

December 11, 2000

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Name

Textbook/Notes used: ____

Directions: Be sure to include in-line citations, including page numbers if appropriate, every time you use a text or notes or technology. **Only write on one side of each page.**

The Problems: Do any 7 of the 9.

1. Let ϕ and ψ be two homomorphisms from a group G to another group G' and let $H \subset G$ be the subset $\{x \in G : \phi(x) = \psi(x)\}$. Prove or disprove: H is a subgroup of G.

2. Use a group action to compute the order of the group of symmetries of a cube, when orientation-reversing symmetries such as reflections in planes, as well as rotations, are allowed. Be sure to specify what set S you are using.

3. Let G be a group, H a subgroup, S the set of left and right cosets of H, and conjugation the action of G on S. Prove the orbit of the left coset gH contains the right coset Hg.

4. Let S be a set on which a group G operates. Let $H = \{g \in G : gs = s \text{ for all } s \in S\}$. Prove H is a normal subgroup of G.

5. Let $\phi : G \to G'$ be an onto homomorphism and let N be a normal subgroup of G. Prove $\phi(N)$ is a normal subgroup of G'.

6. Prove a group of even order must contain an element of order 2.

7. The following patterns represent small portions of two tilings of the infinite plane. Circle one of the following patterns and let G be the group of symmetries of that tiling. Determine the point group of G.

8. Let H be a subgroup of a group G. Prove or disprove: The normalizer $N(H) = \{g \in G : gHg^{-1} = H\}$ of H in G is a normal subgroup of the group G.

9. We say a group action of G on a set S is **faithful** if the only element of G which fixes every element of S is the identity. That is,
$$(gs = s \;\forall s \in S) \Rightarrow (g = e).$$

Let G be the dihedral group of symmetries of a square.

(a) Is the action of G on the vertices a faithful action?

(b) is the action of G on the diagonals a faithful action?